UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

JC13 11_9709_12/RP

UNIVERSITY of CAMBRIDGE
International Examinations

1 Given that $\cos x = p$, where x is an acute angle in degrees, find, in terms of p,

(i) $\sin x$, [1]

(ii) $\tan x$, [1]

(iii) $\tan(90° - x)$. [1]

2

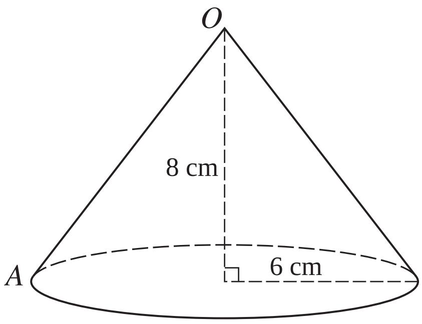


Fig. 1

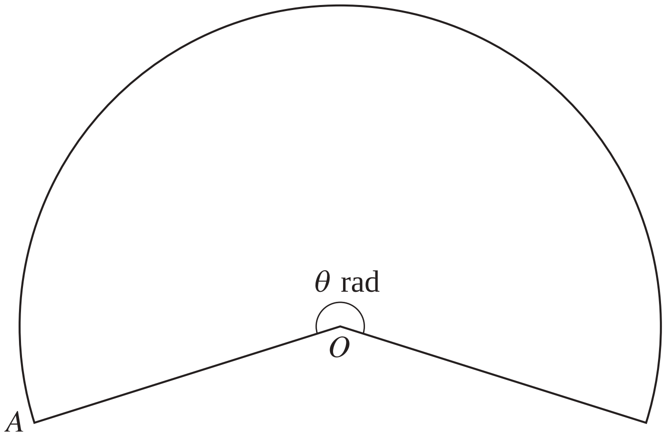


Fig. 2

Fig. 1 shows a hollow cone with no base, made of paper. The radius of the cone is 6 cm and the height is 8 cm. The paper is cut from A to O and opened out to form the sector shown in Fig. 2. The circular bottom edge of the cone in Fig. 1 becomes the arc of the sector in Fig. 2. The angle of the sector is θ radians. Calculate

(i) the value of θ, [4]

(ii) the area of paper needed to make the cone. [2]

3 The equation of a curve is $y = \dfrac{2}{\sqrt{(5x-6)}}$.

(i) Find the gradient of the curve at the point where $x = 2$. [3]

(ii) Find $\displaystyle\int \frac{2}{\sqrt{(5x-6)}}\,dx$ and hence evaluate $\displaystyle\int_2^3 \frac{2}{\sqrt{(5x-6)}}\,dx$. [4]

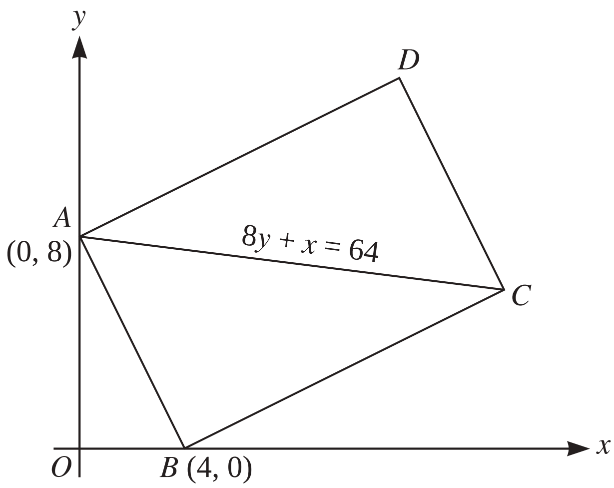
4 Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} \quad \text{and} \quad \overrightarrow{OB} = 4\mathbf{i} + p\mathbf{k}.$$

(i) In the case where $p = 6$, find the unit vector in the direction of $\overrightarrow{AB}$. [3]

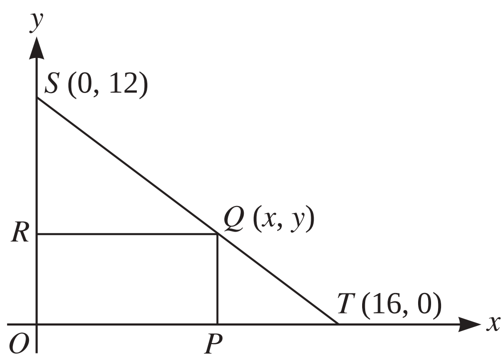
(ii) Find the values of p for which angle $AOB = \cos^{-1}\left(\frac{1}{5}\right)$. [4]

5



The diagram shows a rectangle $ABCD$ in which point A is $(0, 8)$ and point B is $(4, 0)$. The diagonal AC has equation $8y + x = 64$. Find, by calculation, the coordinates of C and D. [7]

6



In the diagram, S is the point $(0, 12)$ and T is the point $(16, 0)$. The point Q lies on ST, between S and T, and has coordinates (x, y). The points P and R lie on the x-axis and y-axis respectively and $OPQR$ is a rectangle.

(i) Show that the area, A, of the rectangle $OPQR$ is given by $A = 12x - \frac{3}{4}x^2$. [3]

(ii) Given that x can vary, find the stationary value of A and determine its nature. [4]

7 **(a)** An athlete runs the first mile of a marathon in 5 minutes. His speed reduces in such a way that each mile takes 12 seconds longer than the preceding mile.

(i) Given that the nth mile takes 9 minutes, find the value of n. [2]

(ii) Assuming that the length of the marathon is 26 miles, find the total time, in hours and minutes, to complete the marathon. [2]

(b) The second and third terms of a geometric progression are 48 and 32 respectively. Find the sum to infinity of the progression. [4]

[Questions 8, 9 and 10 are printed on the next page.]

8 A function f is defined by $f : x \mapsto 3\cos x - 2$ for $0 \leqslant x \leqslant 2\pi$.

(i) Solve the equation $f(x) = 0$. [3]

(ii) Find the range of f. [2]

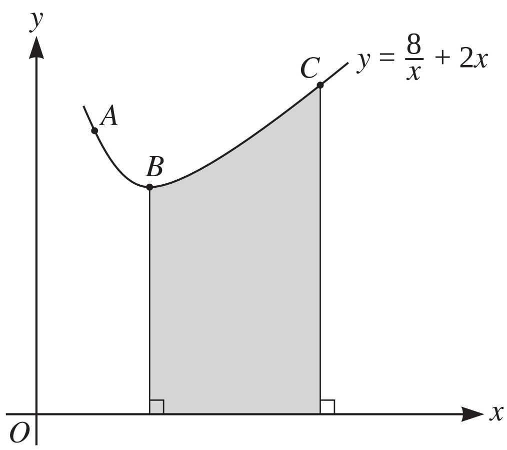
(iii) Sketch the graph of $y = f(x)$. [2]

A function g is defined by $g : x \mapsto 3\cos x - 2$ for $0 \leqslant x \leqslant k$.

(iv) State the maximum value of k for which g has an inverse. [1]

(v) Obtain an expression for $g^{-1}(x)$. [2]

9



The diagram shows part of the curve $y = \frac{8}{x} + 2x$ and three points A, B and C on the curve with x-coordinates 1, 2 and 5 respectively.

(i) A point P moves along the curve in such a way that its x-coordinate increases at a constant rate of 0.04 units per second. Find the rate at which the y-coordinate of P is changing as P passes through A. [4]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis. [6]

10 A curve has equation $y = 2x^2 - 3x$.

(i) Find the set of values of x for which $y > 9$. [3]

(ii) Express $2x^2 - 3x$ in the form $a(x + b)^2 + c$, where a, b and c are constants, and state the coordinates of the vertex of the curve. [4]

The functions f and g are defined for all real values of x by

$$f(x) = 2x^2 - 3x \quad \text{and} \quad g(x) = 3x + k,$$

where k is a constant.

(iii) Find the value of k for which the equation $gf(x) = 0$ has equal roots. [3]